

1 Attorney Docket No. 79740

2
3 IMPROVED FUZZY LOGIC PROCESSING SYSTEM
4 AND METHOD WITH UNCERTAIN INPUT
5

6 The present invention is a continuation-in-part of U.S.
7 Application Serial No. 09/246,208, filed January 20, 1999.
8

9 STATEMENT OF THE GOVERNMENT INTEREST

10 The invention described herein may be manufactured and used
11 by or for the Government of the United States of America for
12 Governmental purposes without the payment of any royalties
13 thereon or therefore.

14
15 BACKGROUND OF THE INVENTION

16 (1) Field of the Invention

17 The present invention relates generally to decision support
18 systems based on fuzzy logic and, more specifically, to a fuzzy
19 logic information processing system such as a control system and
20 method capable of handling uncertainty in the input data with
21 improved computational efficiency.

1 (2) Description of the Prior Art

2 Other than my previous patent application referenced above,
3 traditional fuzzy logic systems do not have the ability to
4 process inexact input data e.g., measurements that are
5 imprecise, or information that has attendant uncertainty. My
6 previous patent application teaches (i) an enhanced fuzzy system
7 and (ii) a novel inferencing method for handling input data
8 uncertainty in a fuzzy logic framework. While the embodiment of
9 a fuzzy logic information processing system disclosed in my
10 previous patent application does handle inexact input data, some
11 usages thereof may be computationally demanding, and may
12 sometimes be impractical for systems of high dimension (3 or
13 greater).

14 Except as taught in my previous application referenced
15 above, methods for data integration and decision support in
16 submarine combat systems do not adequately account for
17 uncertainty in the input data in an automated fashion. Instead
18 they rely heavily on operator manipulation and human
19 interpretation.

20 Combat system information processing entails the
21 integration of data from diverse sources for tactical picture
22 generation and maintenance, situation assessment and planning,
23 and allocation/control of resources. On the other hand, in many

1 systems the amount and flow of input data for integration has
2 been rapidly increasing in recent years. Thus, the cost of more
3 human operators for manning purposes is also rapidly increasing.
4 It is anticipated that advances in sensor technology will
5 continue to offer more possibilities in gathering related both
6 acoustic and non-acoustic streams from organic as well as off-
7 board sources, environmental and kinematic descriptors,
8 intelligence reports and sensor characteristics. The combat
9 system of the future therefore requires the ability to
10 automatically manage uncertainty in areas such as uncertain data
11 input. Uncertainty management for handling uncertainty in the
12 input data remains an outstanding technical issue and
13 constitutes a significant Navy problem as well as a scientific
14 and industrial challenge.

15 Uncertainty refers to being in a condition of doubt. The
16 is contrasted to a condition of certainty or being definite,
17 known, or specific. In an information processing context,
18 uncertainty can be thought of as having a lack of definitive
19 knowledge necessary to describe the process. Uncertainty in the
20 input may result due to many causes including but certainly not
21 limited to sensor noise, gaps in sensor information, sensor
22 bias, inadequate number or placement of sensors, transmission
23 noise or limitations, "one-time only" availability, and the

1 like. While most signals are measured within a tolerance, e.g.,
2 ten volts plus or minus one hundred micro volts, an uncertain
3 signal is not known within the normal tolerances and may be so
4 uncertain that normally used sensor tolerances are meaningless.
5 Thus, while a tolerance of one hundred micro volts might be an
6 accepted tolerance for an accurate signal in a particular
7 application, an uncertain signal might vary by several volts or
8 by more than one thousand times the normal accepted tolerance
9 for the signal thus making the signal quite uncertain in a
10 particular application. Thus, a known or definite signal might
11 be 10 volts, an uncertain signal might be representable only as
12 a possible value between 8 and 12 volts. As another example of
13 uncertain input, a sonar system working in a multipath
14 environment may send out a sonar pulse and receive two or three
15 sonar pulses in return. All three sonar pulses may be received
16 within a time frame that would present reasonable
17 distance/direction information for receipt from the intended
18 target. Therefore, there is uncertainty associated with each
19 returned sonar pulse. As another example, it may be possible to
20 get an uncertain value immediately where a decision for action
21 may need to be taken now where in time a more certain value will
22 be available. This situation may typically arise in a target
23 motion analysis where a fundamental property of bearings-only

1 target motion analysis is that contact range is not observable
2 for a single-leg ownship motion wherein a leg is defined as a
3 time interval of constant platform velocity. The range becomes
4 observable only after an ownship maneuver followed by a second
5 leg of motion that therefore introduces a time-latency in the
6 estimation process owing to the necessity of collecting
7 sufficient data on all legs of motion. Thus, there are many
8 different scenarios of types of uncertainty that will depend on
9 each different situation.

10 As a general matter, an information processing system such
11 as a combat control system or other typical control system will
12 produce one or more specific or definite control signals in
13 response to the input data. A representative example might
14 include a tactical picture display that might show a submarine
15 in relation to other targets. Another example might include a
16 control for a motor to adjust rudder position. This is also
17 true of a fuzzy logic-based control system. Fuzzy logic control
18 systems have been employed successfully employed in various
19 applications. Moreover, fuzzy logic controllers have been
20 successfully applied and demonstrated in underwater combat
21 control systems such as, for example, a conditioned fuzzy logic
22 controller for an acoustic vehicle intercept guidance system.

1 A prior art fuzzy inference system has three basic
2 components. The fuzzifier converts discrete or crisp input
3 numbers to fuzzy logic membership values that describe a
4 qualitative description of the discrete input in semantic terms.
5 For instance, a numerical sensor value such as might be produced
6 from a sensor voltage might be converted from its discrete,
7 known, or specific value to a fuzzy logic member ship value that
8 is more qualitative, e.g., low, medium, or high. The output of
9 the fuzzifier is represented in the membership values, and
10 comprises the fuzzy logic input membership values. The
11 fuzzifier is not designed to handle an input that is inexact
12 and/or which has a possibility of varying throughout a range of
13 values. Traditional usage of this structure has been in
14 control systems applications, where errors in the input are
15 typically ignored. The ill-effects of any error are (hopefully)
16 mitigated by a rapid update-rate; i.e., new measurements or
17 updated information are provided in a time-frame that is short
18 compared to the dynamics of the system to be controlled.
19 Uncertain input data is a problem in data integration and
20 decision support applications (such as submarine combat control)
21 where the measurements have a very slow update-rate or the
22 information is of 'one-time' only availability.

1 The input membership values are used by an inference
2 engine. The inference engine creates a knowledge base from
3 rules that permit an inference and subsequent aggregation of all
4 the output membership functions from the rules that are
5 triggered by the fuzzy logic input membership values. Thus,
6 the inference engine maps the fuzzy logic input membership
7 values to a single fuzzy output set based on applicable rules
8 from the knowledge base.

9 The defuzzifier converts the fuzzy output set to a crisp,
10 discrete, particular output value for subsequent usage, e.g.,
11 the controller output in a feedback system. That is, given an
12 exact number $x=a$ as input, the output is an exact number $y=b$.
13 The crisp output is representative of the fuzzy output set and
14 might be analogous to the expected value in a probability
15 distribution.

16 In summary, a conventional fuzzy system does not have the
17 mechanism to handle an uncertain input, yet such inputs are
18 typically encountered in practice, e.g., data integration for
19 submarine combat control. Simply taking an average, making an
20 estimate, or calculating a normal value and using the discrete
21 value so determined as input to the fuzzy logic inference system
22 will limit the information that is available about the

1 uncertainties, and thereby reduce the likelihood of making the
2 best possible decision.

3 Patents that show attempts to solve the above and other
4 problems are as follows:

5 U.S. Patent No. 5,926,802, issued July 20, 1999, to Manfred
6 Menke, discloses a method in which several rules having an
7 identical conditional part and different consequence parts are
8 processed more quickly in that the fuzzification and at least
9 parts of the inference formation are carried out only once, and
10 in which the storage capacity of a knowledge base memory can be
11 reduced, since all identical conditional parts need be stored
12 only once.

13 U.S. Patent No. 5,809,486, issued September 15, 1998, to
14 Antonino Cuce, discloses an invention that relates to a fuzzy
15 processor having an input X for at least a plurality of input
16 variables X_i and an output U for one or more output results U_k ,
17 and including a fuzzyfication unit FU having an input coupled
18 to the input X, a fuzzy rule processing unit RU having an input
19 coupled to the output of the fuzzyfication unit FU, and a
20 defuzzyfication unit DU having an input coupled to the output of
21 the processing unit CU and an output coupled to said output U,
22 wherein the output of the defuzzyfication unit DU is coupled to

1 the input of the fuzzyfication unit FU and/or to the input of
2 the processing unit RU.

3 U.S. Patent No. 5,710,867, issued January 20, 1998, to
4 Giacalone et al., discloses a method and system for processing a
5 plurality of fuzzy logic rules. The system includes a plurality
6 of fuzzy logic lines, each fuzzy logic line corresponding to one
7 of the fuzzy logic rules and including a calculating device.
8 Each calculating device has an input terminal for receiving a
9 series of weights and an output terminal for outputting an
10 overall truth value according to the received series of weights
11 and at least one logical operator of the fuzzy logic rule
12 corresponding to the fuzzy logic line. The system further
13 includes processing circuitry coupled to each fuzzy logic line,
14 for receiving the overall truth value from each line, and
15 outputting a fuzzy logic value according to the received overall
16 truth values.

17 U.S. Patent No. 5,677,996, issued October 14, 1997, to Lee
18 et al., discloses a fuzzy computer which provides high speed
19 fuzzy processing by executing fuzzy information processing in
20 parallel. The fuzzy computer operates according to programs and
21 data downloaded from a host system, and executes fuzzy
22 information processing in parallel to process this information
23 at high speed. The fuzzy computer comprises a single fuzzy

1 processing controller which downloads and designates to control
2 memory operational programs from the host system, starts fuzzy
3 operations according to signals inputted to an instruction
4 register, and informs the host system of its processing status
5 through a status register. A plurality of fuzzy processing
6 elements are connected in parallel to the single fuzzy
7 processing controller through system buses, download fuzzy data
8 from the host system to each built-in knowledge base and perform
9 respective functions in parallel according to control signals
10 from the single fuzzy processing controller.

11 U.S. Patent No. 5,579,439, issued November 26, 1996, to
12 Emdadur R. Khan, discloses a fuzzy logic design generator for
13 providing a fuzzy logic design for an intelligent controller in
14 a plant control system, including an artificial neural network
15 for generating fuzzy logic rules and membership functions data.
16 These fuzzy logic rules and membership functions data can be
17 stored for use in a fuzzy logic system for neural network based
18 fuzzy antecedent processing, rule evaluation and
19 defuzzification, thereby avoiding heuristics associated with
20 conventional fuzzy logic algorithms. The neural network, used
21 as a fuzzy rule generator to generate fuzzy logic rules and
22 membership functions for the system's plant controller, is a
23 multilayered feed-forward neural network based upon a modified

1 version of a back-propagation neural network and learns the
2 system behavior in accordance with input and output data and
3 then maps the acquired knowledge into a new non-heuristic fuzzy
4 logic system. Interlayer weights of the neural network are
5 mapped into fuzzy logic rules and membership functions.
6 Antecedent processing is performed according to a weighted
7 product of the antecedents. One layer of the neural network is
8 used for performing rule evaluation and defuzzification.
9 However, invention is still conventional in the sense that the
10 inputs need to be exactly specified. U.S. Patent No. 5,524,179,
11 issued June 4, 1996, to Masae Kanda, discloses a fuzzy inference
12 apparatus based on fuzzy rules represented by an if-then
13 notation, comprising a CPU having a software for obtaining
14 grades of if-part membership functions with respect to input
15 variables for each rule and a hardware for obtaining a truth
16 value of the if-part of each rule, a truth value of the then-
17 part of each rule, and an inference result. The hardware
18 comprises a predetermined number of then-part membership
19 function calculators for respectively storing the then-part
20 membership functions defined in the then-part of the rules,
21 minimum calculators respectively connected to the then-part
22 membership function calculators, and a maximum calculation
23 circuit connected to all the then-part membership function

1 calculators. The CPU supplies the grades of the if-part
2 membership functions for each rule to one of the minimum
3 calculators which is connected to the then-part membership
4 function calculator storing the then-part membership function
5 defined in the then-part of the rule. The CPU outputs a
6 predetermined number of grade regardless of the number of the
7 input variables defined in the rule. The grade of the
8 membership function is set to 1 where the input variable is not
9 defined in the rule. However, this invention suffers from the
10 basic limitation of conventional fuzzy system in that the input
11 variables need to be precisely specified. There is no mechanism
12 to process inexact input data, e.g., measurements that are
13 imprecise or information with attendant uncertainty.

14 A publication, the Naval Undersea Warfare Center Division
15 Newport, Newport RI, NUWC-NPT Technical Report 10,876, dated 20
16 January 1998, entitled "Fuzzy Logic-Based Inferencing in the
17 Presence of Input Data Uncertainty," authored by Chidambar
18 Ganesh, describes the use of a fuzzy inference machine to take
19 an input which includes uncertain data item(s) and provides
20 fuzzy inference output which represents this uncertainty.
21 However, the computation process which the publication teaches
22 are very numerically intensive and require very large amounts of

1 dynamic random access memory (RAM) storage associated with a
2 processor.

3 In view of the above-cited prior art which does not show
4 how to handle uncertain data, there remains a need for a fuzzy
5 logic-based information processing system that can not only
6 handle uncertain input but do so with improved computational
7 efficiency. Those skilled in the art will appreciate the
8 present invention that addresses the above and other problems.

9
10 SUMMARY OF THE INVENTION

11 Accordingly, it is an object of the present invention to
12 provide an improved system and method for processing data using
13 fuzzy logic with increased computational efficiency.

14 It is another object of the present invention to provide
15 such an improved fuzzy logic system that is capable of
16 processing either certain or uncertain data.

17 It is yet another object of the present invention to
18 provide an improved fuzzy logic-based control system and method.

19 It is yet another object of the present invention to
20 provide an improved fuzzy logic-based tactical picture and
21 decision support system.

1 These and other objects, features, and advantages of the
2 present invention will become apparent from the drawings, the
3 descriptions given herein, and the appended claims.

4 In accordance with the present invention, a fuzzy inference
5 system for processing information including uncertain
6 information is disclosed. At least one input provides
7 information that is indicative of one or more physical
8 phenomena. The input is representable by an input set that
9 describes a range of possible values related to the one or more
10 physical phenomena. When a precise value for the input is not
11 available such that the input value is uncertain, then an input
12 data set is representable mathematically by a first map of
13 possible values related to the one or more physical phenomena.

14 The fuzzy inference system may comprise elements, such as
15 for instance, a rule decomposer that preferably includes a
16 plurality of rules. Each of the rules may be utilized for
17 producing an output in response to the plurality of inputs to
18 thereby produce a plurality of rule decomposer outputs. Other
19 elements may include a union operator for determining an
20 intersection of the plurality of rule decomposer outputs to
21 produce an fuzzy inference system output.

1 In one presently preferred embodiment, each of the plurality of
2 rules may be of an *IF - THEN* format. The fuzzy inference system
3 output signal may be used as a control signal.

4 In one embodiment, a dynamic RAM storage may be utilized
5 for storing precomputed outputs of the rule decomposer.

6 In another embodiment, the system may provide a system
7 control signal, and in yet another embodiment it may provide a
8 tactical picture.

9 A method in accord with the present invention for a fuzzy
10 inference system that utilizes uncertain input data may comprise
11 steps such as providing a plurality of rules, producing a
12 plurality of rule outputs in response to the uncertain input
13 data, and inferring an output of the fuzzy inference system by
14 determining a conjunction of the plurality of rule outputs.

15 Other steps may include operating on the input set of
16 possible values to produce a rule output for each of the
17 plurality of rules and/or precalculating a rule output. If
18 precalculations are made, then a preferred embodiment may
19 include storing a result of the step of precalculating in RAM
20 memory. In one embodiment the method may comprise producing a
21 tactical picture that incorporates the output from the fuzzy
22 inference system. Another method for a fuzzy inference system
23 in accord with the present invention for utilizing uncertain

1 input data that has an uncertain value contained within an input
2 set of possible values such that the uncertain input data may be
3 associated with one or more physical phenomena, may comprise
4 steps such as, for instance, producing a plurality of one
5 dimensional solutions in response to the uncertain input data,
6 and inferring an output of the fuzzy inference system by
7 determining a conjunction of the one dimensional solutions.

8 The method may further comprise producing the plurality of
9 one dimensional solutions from a plurality of rules. Moreover,
10 the method may comprise producing a respective one dimensional
11 solution from a respective rule.
12
13

14 BRIEF DESCRIPTION OF THE DRAWINGS

15 A more complete understanding of the invention and many of
16 the attendant advantages thereto will be readily appreciated as
17 the same becomes better understood by reference to the following
18 detailed description when considered in conjunction with the
19 accompanying drawings wherein corresponding reference characters
20 indicate corresponding parts throughout several views of the
21 drawings and wherein:

22 FIG. 1 is a block diagram of an embodiment of a fuzzy logic
system capable of processing either certain or uncertain data

1 which is precursory of the fuzzy inference system architecture
2 of FIG. 10;

3 FIG. 2 is a block diagram of a fuzzy logic inference system
4 as might be used in the fuzzy logic information processing
5 system of FIG. 1;

6 FIG. 3 is a mathematical representation of the data that is
7 illustrative of data operated upon by the inference system of
8 FIG. 2, and that is uncertain wherein it is known only that the
9 actual value is within the shaded region;

10 FIG. 4 is map illustrative of rules involved in the
11 operation of the inference system of FIG. 2:

12 FIG. 5 is an arbitrary graph descriptive of a bell or
13 Gaussian curve illustrative of a set of possible values for the
14 input for which any particular value in the set is uncertain,
15 which may be involved in the operation of the inference system
16 of FIG. 2;

17 FIG. 6 is a map of the uncertain data of FIG. 5 that has
18 been extended into another dimension by an extensor or
19 cylindrical extensor in accord with the present invention:

20 FIG. 7 is a map of the combination of the map of FIG. 5 and
21 the map of FIG. 6;

22 FIG. 8 is an output for the information processing system
23 of FIG. 1 that shows the output in the desired terms and also

1 compares a fixed value input with the results of an uncertain
2 input;

3 FIG. 9 discloses one of many possible tactical displays in
4 which the output of the information processing system of FIG. 1
5 may be used; and

6 FIG. 10 depicts a fuzzy inference system architecture with
7 improved computational capabilities based on a rule
8 decomposition in accord with the present invention.

9
10 DESCRIPTION OF THE PREFERRED EMBODIMENTS

11 While the invention described in my previous patent
12 application referenced hereinbefore is capable of using inexact
13 input data, the computation can be very numerically intensive,
14 and is an exponential function of the number of inputs and the
15 grid-resolution employed.

16 There is first hereinafter provided, with reference to
17 FIGS. 1-9, a description of the embodiment of fuzzy logic
18 processing system disclosed in my previous patent application.
19 There is then provided, with reference to FIG. 10, a description
20 of a fuzzy inference system architecture based upon Rule
21 Decomposition in accordance with the present invention, which is
22 less numerically intensive, with the numerical intensity only

1 growing as a linear function of such inputs and grid-
2 resolutions.

3 Referring now to the drawings and more specifically to FIG.
4 1, a fuzzy logic processing system 10 detects stimulus 11, and
5 the latter's characteristics are relayed to output 36, which
6 uses fuzzy logic when uncertainties related to stimulus 11
7 and/or other aspects of the input exist. Thus, the problem
8 involves uncertainties that propagate through system 10.
9 Stimulus 11, which may be one or more physical phenomena of some
10 type such as propeller rotation frequency or pattern and may
11 produce one or more of signals 12, 14 and 16. One or more of
12 signals 12, 14, and 16 are received by remote organic sensor 18,
13 organic sensor 20, and/or off board sensor 22, respectively.
14 Signals 12, 14 and 16 could be descriptive or related to a
15 natural physical phenomena such as color or a reflection of some
16 kind like radar or sonar. Remote organic sensor 18 and organic
17 sensor 20 use in this case the definition of organic as being
18 associated with the body or craft that the sensors are either
19 attached to or are in control of. Organic sensors can therefore
20 include a remotely launched probe, sonar, radar, or any type of
21 device which detects physical phenomena such as, for instance,
22 detection of another object for tracking purposes. Off board
23 sensor 22 is independent of the body or craft and can transmit

1 the data relating to stimulus 11 which, as discussed above, may
2 represent numerous different types of stimuli. The data
3 collected by sensors 18, 20 and/or 22 are transmitted through
4 respective possible transmission paths 24, 26 and 28. These
5 paths may have physical imperfections which can cause gaps in
6 the data, as depicted by uncertainty 30. Uncertainty 30 could
7 arise in many ways and at many places in the processing system
8 and is pictured in a specific place only for convenience. For
9 instance, sensors 18, 20 and 22 may not have detected all
10 signals or have received erroneous information, thus more
11 uncertainty is present. There may be a time lag problem as
12 discussed subsequently. It will be understood by those skilled
13 in the art that there are virtually an infinite number of
14 reasons that can arise to cause uncertainty within the input
15 data. Thus, the result is that data with inherent
16 uncertainties is received by fuzzy inference system 32 from one
17 or more of signal paths 24, 26, and/or 28. While prior art
18 fuzzy inference systems cannot handle uncertain data, the
19 inference system of my previous patent application can handle
20 uncertain or certain data, as explained in more detail
21 hereinafter, using rules 34 within fuzzy inference system 32 to
22 yield output 36, that may be used for producing a tactical

1 picture, guidance presets, motor control, decision support, and
2 the like.

3 The fuzzy inference system 32 disclosed in my previous
4 patent application is shown in greater detail in FIG. 2. The
5 collective data sent over paths 24, 26 and/or 28 detected by
6 sensors 18, 20 and 22 contains uncertainty as represented
7 graphically by the region of fuzzy input 38 wherein it is
8 uncertain what the precise value of fuzzy input 38 is. Fuzzy
9 input 38 is mathematically represented by $\mu_a(x)$, with collective
10 data that varies about the line $x = a$. Thus, whereas the prior
11 art fuzzy inference systems would require a precise input, such
12 as $x = a$, system 32 can handle the uncertain input where it is
13 known only that input x varies about a to form a set of possible
14 values described by the function $\mu_a(x)$. The region or set of
15 values described by $\mu_a(x)$ is arbitrarily selected in the present
16 example and could take on many different forms or shapes. For
17 the present explanation purposes, a Gaussian bell type curve
18 distribution describes the region of possible values for the
19 input as shown by FIG. 5. The input could be in the form of a
20 square wave, pulse, multiple sections, or other shapes. While
21 the example given herein uses a numerical characterization of
22 uncertain data, it will be understood that the present invention
23 is not limited to numerical characterization and could also be

1 used with other types of symbolic characterizations of data as
2 might be used for the particular problem to be solved.

3 Fuzzy input 38 is received by fuzzy inference system 32 and
4 operated on by extensor or cylindrical extensor 40. Cylindrical
5 extensor 40, in the present example, operates on fuzzy input 38
6 to provide an extension of fuzzy input 38 in the x, y plane to
7 form extension 41, as represented by

$$\mu_a(x, y) = \mu_a(x) \quad \forall y. \quad (1)$$

9 Extension 41 might be graphically described as adding an
10 extra dimension or, in this case, a third dimension. However,
11 it will be understood that depending on the complexity of the
12 problem many dimensions may be involved so that a visually
13 understandable picture of an extended bell curve as might be
14 exemplified by FIG. 6 may not always be available for every
15 problem. The modified data or extension 41 is made available
16 for fuzzy mapping section 42 for further operation. Fuzzy
17 section 42 is derived from rules 34 and is mathematically
18 represented by $F(x, y)$. Fuzzy mapping section 42 operates on
19 extension 41 to yet again amend the data to form the conditioned
20 data or surface 43 or $F_c(x, y)$ which might be visualized as shown
21 in FIG. 7. The whole operation performed by fuzzy mapping 42
22 can be described as

$$F_c(x, y) = \min[F(x, y), \mu_a(x, y)]. \quad (2)$$

1 In the example of the present case, this can be verbally,
 2 symbolically, or linguistically restated as the graphical
 3 intersections of rules 34 and the output of cylindrical extensor
 4 40 to form conditioned surface 43. Now retro-projector 44
 5 receives the $F_c(x,y)$ or conditioned surface 43 and transforms it
 6 into fuzzy output 46 or $M_{\mu_a}(y)$ as might be graphically
 7 represented as shown in FIG. 8. This process can be symbolized
 8 by the formula

$$M_{\mu_a}(y) = \max_{\forall x} [F_c(x,y)]. \quad (3)$$

9
 10 The process of the above equation may be visually or
 11 graphically described as projecting the data output from fuzzy
 12 mapping 42 onto the y plane, resulting again in two dimensional
 13 data having the desired output units as shown, for example in
 14 FIG. 8.

15 FIG. 3 provides an example that graphically depicts the
 16 operation of fuzzy inference system 32 in a particular case as
 17 discussed below. Point 48 represents the value $x = a$, an exact
 18 value. While a prior art fuzzy logic inference system would
 19 require such an exact value, a crisp input value is not known in
 20 the present example so that shaded region 52 generalizes x as
 21 being in that area, defined by the function $\mu_a(x)$. Had $x = a$
 22 been known, point 50 would represent $y = b$, an exact value as
 23 might be described using a prior art fuzzy logic inference

1 system. The cylindrical extensor extends the function $\mu_a(x)$ into
2 another dimension with

$$\mu_a(x, y) = \mu_a(x) \quad \forall y, \quad (4)$$

4 as discussed above. The input data now is uncertain but the set
5 of possible values has the form $\mu_a(x, y)$ represented by the area
6 between lines 49 and 51. The rules are graphically described as
7 the value between lines 54 and 56 and represented by $F(x, y)$.

8 Shaded region 58 is generated by the cylindrical extensor output
9 and the rules to form, represented by $F_c(x, y)$. In other words,
10 the area defined by lines 49, 51, 54 and 56. Mathematically,
11 $F_c(x, y) = F(x, y) \circ \mu(x, y)$, with \circ being a fuzzy composition
12 operation. In comparison, had exact values been known as
13 required in a prior art fuzzy logic inference system, line 50
14 would accurately depict this output. Finally, the data is
15 transformed back into usable form by projecting the graphical
16 image into the desired units by removing a dimension,

17 represented by line 62. This is the fuzzy output $M_{\mu a}(y)$, given
18 by

$$M_{\mu a}(y) = \max_{\forall x} [F_c(x, y)]. \quad (5)$$

20 FIGS. 4-9 give a possible example of a step-by-step
21 graphical representation of the process of fuzzy inference
22 system 32 of a type for use in an information and decision
23 system, or for use as a stage in a decision process. Such

1 systems or decision processes are used in naval undersea
2 warfare, for example to provide with a submarine the display
3 depicted in FIG. 9 showing position of an acoustic contact
4 relative to ownship position. Where the information concerning
5 the position of the acoustic contact is uncertain due to
6 uncertain physical phenomenon affecting acoustic signal
7 propagation, the display of FIG. 9 assists the submarine's
8 commander in making combat control decisions in the presence of
9 uncertain physical phenomenon. As will be explained below, the
10 display of FIG. 9 is derived from output 46 of system 32. It
11 will be understood that this example is given for explanatory
12 purposes only. Another example - this time in providing an
13 input to a control system - is the application of output 46 from
14 system 32 as a control system input signal for a homing guidance
15 control system of a self-guided underwater missile to provide
16 improved kill capability thereof. The invention is not intended
17 to be limited by these examples and may be used in different
18 applications and conditions, including, but not limited to:
19 medical, industrial, marine, broadly any form of warfare,
20 exploration, and numerous other settings.

21 Referring now to FIG. 4 and subsequent figures, the axes x ,
22 y and z are defined as class axis 66, membership axis 68 and
23 speed axis 74, respectively. The example is based upon a two

1 rule, one input, one output fuzzy inference system, which will
2 provide target speed based upon detected class. The target will
3 have a higher speed if it has a nuclear engine than if it has a
4 diesel engine. Class axis 66 ranges from zero to one, with zero
5 denoting diesel and one representing nuclear. Membership axis
6 68 also ranges from zero to one, with zero designating no
7 membership and one indicating one hundred percent membership in
8 the specified class. Therefore, a target is diesel if class
9 equals zero and membership equals one, whereas a target is
10 nuclear if class equals one and membership equals one. Speed
11 axis 74 ranges from zero to forty, or the range of speed of the
12 target in knots. A nuclear engine target is more likely to be
13 traveling in the range of 15 to 30 knots as compared with a
14 diesel engine target with a speed more likely in the range of 5
15 to 15 knots. The rules can be summed up as the following: (1)
16 if class is diesel, then speed is low and (2) if class is
17 nuclear then speed is high, as depicted by the three dimensional
18 rule graph 70 represented by the function $F(x,y)$. The rules
19 for the present example are graphically displayed as a set, map,
20 figure, or curve as shown in FIG. 4. It will be understood that
21 while the map or set of values indicated may be easily
22 visualized in three dimensions as in the example of FIG. 4,
23 other systems of rules, inputs, and/or outputs may use many

1 dimensional or n-dimensional maps that are not so easily
2 visualized.

3 The example further continues with the fact that sensors and/or
4 other intelligence have detected a target with class equal to
5 about 0.3 with some amount of uncertainty. The sensors used for
6 this purpose might conceivably be sonar intelligence that
7 detects a propeller speed or type of sound or the like. The
8 result of the detected input is graphically depicted in FIG. 5
9 with an input that is uncertain but has possible values about
10 point 64, representing $x=0.3$. For this example, a Gaussian
11 membership function, graph, map or set 72 describes the fuzzy
12 input. The target is not described exactly in class equal to
13 0.3, but rather is described as a possible value within the area
14 under the curve but uncertain as to exactly what that value is.
15 If the input were definite, such that the information was
16 exactly 0.3, then the present invention would also operate to
17 handle that situation in the same manner as discussed below.
18 Function 72 would then simply be a straight line at 0.3. Thus,
19 the present invention is operable with both definite and
20 uncertain data. However assuming the input to be uncertain,
21 then Gaussian membership function 72, for an example only, may
22 have formula equal to

$$\mu_{0.3}(x) = \exp \left[-\frac{(x - 0.3)^2}{2 * 0.05^2} \right] \quad (6)$$

In accord with the system and method of the present invention, Gaussian membership function 72 is extended onto speed axis 74 to make cylindrical extension map, function, set or graph 76 as shown in FIG. 6, depicting the operation of extensor or cylindrical extensor 40. Thus, the input is extended onto at least one dimension characteristic of the rules. Cylindrical extension map, set, function or graph 76, represented by $\mu_{0.3}(x,y)$, is obtained by the mathematical expression discussed above, i.e.,

$$\mu_{0.3}(x,y) = \mu_{0.3}(x) \quad \forall y. \quad (7)$$

Fuzzy mapping section 42 operates on cylindrical extension map, set, or graph 76 and rule map, set, or graph 70. In the present example an intersection is found as the result of this operation that provides map, set, or conditioned surface graph 78 as shown in FIG. 7. Conditioned surface map, set or graph 78, noted as $F_c(x,y)$, was obtained in accord with the equation expressed above, i.e.,

$$F_c(x,y) = \min[F(x,y), \mu(x,y)]. \quad (8)$$

In FIG. 8, conditioned surface graph 78 is fed into the retro-projector where the graph is projected onto speed axis 74 and possibility axis 84. A dimension is deleted in accord with

1 the desired output terms resulting in two dimensional output
2 fuzzy data 80. Interpretation of this graph states that the
3 target is more likely to be traveling about ten knots, but still
4 has the possibility of going about twenty to thirty knots,
5 though this is not as likely. Line 82 is the graph obtained had
6 exact values been taken with $x=0.3$, resulting in the uncertainty
7 obtained using fuzzy input being deleted and never taken into
8 account. Had this been done, as would have been a plausible
9 solution for prior art fuzzy logic inference systems that
10 require an exact input, then the likelihood would have been
11 significantly greater that the speed is in the range of about
12 ten knots. Thus, a decision maker might have been more likely
13 to make a decision that committed too early to the decision,
14 implicitly basing the decision upon an assumption that the
15 target speed is in the range of about ten knots. A better
16 decision would have been to wait as long as possible before
17 committing due to the increased possibility that the speed was
18 in the range of from fifteen to thirty knots when the
19 uncertainty of the input was included in the calculation.

20 In a bearings-only target motion analysis problem it is
21 necessary to estimate contact location and motion parameters
22 using a time series of bearing measurements. A fundamental
23 problem of the bearings-only target motion analysis application

1 is that the contact range is not observable for a single-leg of
2 ownship motion wherein a leg is defined as a time interval of a
3 constant platform, such as a ship or submarine, velocity. A
4 time lag is therefore introduced into the estimation process
5 owing to the necessity of collecting sufficient data on all legs
6 of motion. In some cases rapid estimates are needed even though
7 they may be of poorer quality due to the time delay. Tactical
8 map 86, shown in FIG. 9, is one possible resulting use of the
9 data obtain about the target. The fuzzy characterization of
10 contact speed as discussed above using the fuzzy logic inference
11 system output as shown in FIG. 8 to produce an enhanced area of
12 uncertainty description for the single leg target motion
13 analysis. Tactical map 86 shows in Kyards the target's
14 suspected contact end-point position 96 in relation to observer
15 94. Tactical map 86 also displays area 88, depicted 50%
16 possibility of the target end point being in that area. Area 90
17 depicts 85% confidence, and area 92 denotes 98% confidence of
18 the target being within those borders. For reference, location
19 98 is the actual target end point position. While end-points of
20 the likely tracks are displayed in tactical map 86, the various
21 possible tracks with the most likely track may also be viewed
22 with a colored weighting for the likelihood of the various
23 tracks.

1 The computation of the fuzzy nonlinear mapping $F(x,y)$ as
2 discussed hereinbefore is very numerically intensive, and is a
3 function of the number of inputs and the grid-resolution
4 employed. For instance, the 2-input fuzzy inference system
5 characterizing contact speed takes approximately 45 minutes to
6 compute on a Pentium 100MHz PC system (with grid-size 100).
7 Since this mapping is fixed for a given fuzzy system, it is
8 precomputed off-line and stored prior to actual usage of the
9 fuzzy inference system. A vectorized approach to the fuzzy
10 composition algorithm was devised using the MATLAB computing
11 environment. The resultant system is found to evaluate the
12 fuzzy output in about 10 seconds, thus achieving significant
13 speed-up and realistic computing times.

14 However, memory storage requirements for the fuzzy
15 inference system mapping $F(x,y)$ increase significantly with the
16 number of inputs. The scale-up in dynamic RAM storage required
17 at run-time as the fuzzy system dimension, defined as number of
18 inputs, increases quickly as the number of inputs or dimensions
19 increases. For instance, if a one dimensional or one input
20 system requires 80 KBytes of RAM storage, a two dimensional
21 system may require 8 MBytes of RAM storage, and a three
22 dimensional system may require 800 MBytes of RAM storage.

1 This poses severe limitations on the utility of the fuzzy
2 composition method in a real-time system application, and is
3 impractical for fuzzy systems of dimension 3 or greater. In
4 accordance with the present invention, an efficient
5 computational method for propagation of uncertain inputs through
6 the fuzzy inference system is based on the principle of Rule
7 Decomposition, as presently will be described.

8 Referring now to FIG. 10, a preferred embodiment of the
9 present invention is disclosed. FIG. 10 provides a block
10 diagram of fuzzy system 100 that provides an efficient
11 architecture for propagation of uncertain inputs through a fuzzy
12 system. In fuzzy system 100 of FIG. 10, the fuzzy mapping
13 $F(x,y)$ derives from a knowledge-base of M rules. In a preferred
14 embodiment of the present invention, the fuzzy mapping 46A is
15 expressed as the conjunction of rule outputs or mappings 106
16 from union operator 108 of each individual rule 102 whereby rule
17 decomposer 104 comprises a plurality of individual rules 102 as
18 described in more detail subsequently. Mathematically stated:

$$F(x,y) \equiv R_1(x,y) \cup R_2(x,y) \cup \dots R_M(x,y) . \quad (9)$$

22 Let the l th rule be denoted by $R_l(x,y); l = 1,2,\dots M$. Hence, the
23 fuzzy inference system output fuzzy set is:

$$M_{\mu_a}(y) = F(x, y) \circ \mu_a(x) = \bigcup_{l=1}^M R_l(x, y) \circ \mu_a(x) . \quad (10)$$

Define fuzzy set $\mu_l(y)$ as the output response 106 of a rule 102 such as the rule $R_l(x, y)$ to an input 38 such as fuzzy input $\mu_a(x)$ Then

$$\mu_l(y) = R_l(x, y) \circ \mu_a(x) = S_{\forall x} [T(R_l(x, y) \mu_c(x, y))] . \quad (11)$$

Each rule 102 such as rule $R_l(x, y)$ may define a fuzzy relation of the form *IF (x is F) THEN (y is G)* and may be denoted as

$$R_l(x, y) = T(\mu_F(x), \mu_G(y)) \quad (12)$$

Substituting and simplifying Eq. (12) into Eq. (11) as follows leads to an expression for output response 106 of the l th rule:

$$\mu_l(y) = T(\mu_G(y), S_{\forall x} [T(\mu_F(x), \mu_a(x))]) \quad (13)$$

For *min-max* implementation of the *T*- and *S*-norms, the Eq. (13) can be reduced to

$$\mu_l(y) = \min(\mu_G(y), \mu_H(x_{\text{sup}})), \text{ where}$$

$$\mu_H(x_{\text{sup}}) = \max_{\forall x} \min(\mu_F(x), \mu_a(x)) \quad (14)$$

Finally, the fuzzy inference system output fuzzy set from Eq. (10) is determined via the union operation determining a conjunction of the plurality of individual rules, and is given by

$$M_{\mu_a}(y) = \bigcup_{l=1}^M \mu_l(y) \quad (15)$$

whereby union operator produces the above described output.

In essence, the method of the present invention utilizes Rule Decomposition as a divide-and-conquer strategy that reduces a large computational problem to several tractable sub-problems. If n = system-dimension (number of inputs), then the fuzzy composition $R_1(x, y)$ is in general over the n -dimension space $X_1 \times X_2 \times \dots \times X_n$. The rule decomposition of Eq. (10) has the effect of reducing this n -dimensional problem to n 1-dimensional sub-problems for each rule. Hence, the Rule Decomposition method requires the equivalent of solving $M \times n$ 1-dimensional sub-

1 problems. In this way, the present invention provides
2 computation efficiency.

3 Moreover, if N = grid-dimension, it can be shown that the
4 number of full operations (*flops*) required for Rule
5 Decomposition and Fuzzy Composition are:

$$6 \quad \text{flops}(RD) = [2(n+1)M - 1] * N, \text{ and} \quad (16)$$

$$8 \quad \text{flops}(FC) = 2N^{n+1}. \quad (17)$$

9 Therefore, a plot of computational cost versus system
10 dimension would show that while the computational cost for the
11 Fuzzy Composition system 10 as discussed hereinbefore grows
12 exponentially with system dimension, the method of the present
13 invention utilizing Rule Decomposition techniques increases only
14 linearly and is hence the improved method of choice.

15 Another significant advantage of the method of the present
16 invention utilizing Rule Decomposition relates to the nominal
17 requirement on dynamic memory storage at run-time. At any given
18 time, the memory used is that employed in solving a 1-
19 dimensional sub-problem, together with the associated book-
20 keeping overhead. Computer simulation experiments show that
21 this memory usage has very little impact on system performance
22 and is well within typical PC-application requirements.

1 It will be understood that many additional changes in the
2 details, materials, steps and arrangement of parts, which have
3 been herein described and illustrated in order to explain the
4 nature of the invention, may be made by those skilled in the art
5 within the principle and scope of the invention as expressed in
6 the appended claims.

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